

Ф. И. Методическая конференция
по интенсивным социаль-
но-экономическим нагрузкам и их
последствиям.

Чембогу, Кигали, 9.06.92

EQUATION OF STATE OF DENSE EXPLOSIVES DETONATION PRODUCTS

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Equation of state of dense explosives detonation products is considered. For determining of two functions of density, drawn into equation of state, Jouguet points line equation is deduced and functions of density on this line are considered instead of functions of initial density of explosive. Limitations on front velocity, caloricity and adiabatic index dependencies upon initial density are formulated. Values of equation of state parameters for several explosives are listed.

Key words: Density, Velocity, Pressure, Equation of state, Explosive

Equation of state of explosives detonation products (DP) in the form

$$P = [\gamma(\rho) - 1]\rho E + \varphi(\rho) \quad (1)$$

was considered by several authors. Form of $\gamma(\rho)$ and $\varphi(\rho)$ functions and numerical values of involved parameters determine the area of applicability of constructed equation of state^[1-6].

We shall use reliably determined in laboratory experiments dependencies of normal detonation wave velocity D , particle velocity u behind the front, and explosive caloricity Q upon initial explosive density ρ_0 before detonation wave front.

Conservation laws for strong discontinuity with instantaneous energy release have the following form

$$\begin{cases} \rho(D - U) = \rho_0(D - U_0) \\ P - P_0 = \rho_0(D - U_0)(U - U_0) \\ D = U + C, \quad C = \sqrt{(\partial P / \partial \rho)_s} \\ E - E_0 = 0.5(U - U_0)^2 \\ \quad + P_0(U - U_0) / [\rho_0(D - U_0)] + Q \end{cases} \quad (2)$$

Let's introduce a notion of crystal i. e. maximum possible in normal conditions density ρ_{st} and a notion of normal detonation wave velocity D_s for explosive with this density, and go on to non-dimensional variables:

$$\begin{cases} \Delta = \frac{\rho_0}{\rho_{st}}, & \delta = \frac{P}{\rho_{st}}, & \Pi = \frac{P}{\rho_{st} D_s^2} \\ M = \frac{U}{D_s}, & Z = \frac{C}{D_s}, & W = \frac{D}{D_s} \\ \Phi = \frac{\varphi}{\rho_{st} D_s^2}, & J = \frac{E}{D_s^2}, & K = \frac{Q}{D_s^2} \end{cases} \quad (3)$$

Owing to invariance of (2) relatively to Galilei transform, we consider that $u_0 = 0$. We accept also that explosive pressure and energy are negligible in comparison with P and E in Jouguet points. After passing to variables (3) and in accordance with suppositions made, Equations (1) and (2) will take a form

$$\begin{cases} \delta = W\Delta / (W - M), \\ \Pi = WM\Delta, \\ W = M + Z \end{cases} \quad (4)$$

$$\begin{cases} J = 0.5 M^2 + K, \\ Z^2 = \left[\frac{\partial \Pi}{\partial \delta} \right], \\ \Pi = (\gamma - 1)\delta J + \Phi \end{cases} \quad (5)$$

System of six Equations (4) and (5) containing nine functions of Δ (Π , δ , J , W , M , K , Z , γ , Φ) becomes determined, if we set any three of this functions.

Let's write down adiabatic index $N = (\partial \ln P / \partial \ln \rho)_s$, in non-dimensional form

$$N = Z^2\delta/\Pi \quad (6)$$

We shall exclude now M from (4) and replace Z by N using (6). Then we shall use thermodynamic equation

$$\left[\frac{\partial\Pi}{\partial\delta}\right]_s = \left[\frac{\partial\Pi}{\partial\delta}\right]_J + \frac{\Pi}{\delta^2}\left[\frac{\partial\Pi}{\partial J}\right]_s \quad (7)$$

Let's differentiate (5) and substitute derivatives

$$\begin{aligned} \left[\frac{\partial\Pi}{\partial J}\right]_s &= (\gamma - 1)\delta \\ \left[\frac{d\Pi}{d\delta}\right]_J &= \frac{d\Phi}{d\delta} + (\gamma - 1)J + \delta J \cdot \frac{d\gamma}{d\delta} \end{aligned}$$

into (7) and then $(\partial P/\partial\delta)_s$ into (4). After excluding J and Π in obtained equation, we shall have

$$\begin{aligned} \frac{d\Phi}{d\delta} + \left[\gamma - 1 + \delta \cdot \frac{d\gamma}{d\delta}\right] \left[\frac{W^2 + 2K(N+1)^2}{2(N+1)^2}\right] \\ + \frac{(\gamma - 1)NW^2}{(N+1)^2} - \left[\frac{NW}{N+1}\right]^2 = 0 \quad (8) \end{aligned}$$

All the values on Jouguet points line depend upon Δ . Let's pass from differentiation by δ in (8) to differentiation by Δ . After multiplying (8) by

$$\frac{d\delta}{d\Delta} = \frac{N+1}{N} - \frac{\Delta}{N^2} \cdot \frac{dN}{d\Delta}$$

we come to

$$\begin{aligned} \frac{d\Phi}{d\Delta} + \frac{A}{2N(N+1)} \cdot \frac{d\gamma}{d\Delta} \\ + \frac{(\gamma - 1)A + 2NW^2(\gamma - 1 - N)}{2N(N+1)} \\ \times \left[1 - \frac{\Delta}{N(N+1)} \cdot \frac{dN}{d\Delta}\right] = 0 \quad (9) \end{aligned}$$

where $A = 2K(N+1)^2 + W^2$. With the help of (4) and (5) we can express Φ through W , Δ , N , K and δ

$$\Phi = \frac{\Delta W^2}{N+1} - \frac{(\gamma - 1)\Delta A}{2N(N+1)} \quad (10)$$

Differentiating Φ by Δ in (10) and excluding the derivative from (9) lead us to Jouguet points line equation in the form

$$\begin{aligned} W\Delta(2N - \gamma + 1) \cdot \frac{dW}{d\Delta} + W^2(\gamma - N)N \\ - (\gamma - 1)(N+1)^2\Delta \cdot \frac{dK}{d\Delta} = 0 \quad (11) \end{aligned}$$

This equation contains $W(\Delta)$, $K(\Delta)$, $N(\Delta)$, that can be

measured. Experimental data on $W(\Delta)$ for many explosive fits dependence

$$W \approx \Delta^a \quad (12)$$

where $a=0.7$ with satisfactory preciseness. Substitution (12) into (11) leads to

$$\gamma = 1 + \frac{N \cdot (N - 1 - 2a)}{N - a - (N - 1)^2 \cdot \Delta^{(1-2a)} \cdot dK/d\Delta} \quad (13)$$

Let's introduce some more limitations for $N(\Delta)$ as $K(\Delta)$ functions. We shall demand that $N \rightarrow \gamma \rightarrow \gamma^0$ while $\Delta \rightarrow 0$. There is few data on explosive caloricity in literature and it has considerable spread. We present data from Ref. [1, 3-9] in non-dimensional form and appreciate it with non-dimensional dependencies on Δ , common for different explosives. From the condition of γ finiteness, i. e. condition of simultaneous turning to zero of numerator and denominator in (13), we obtain demands that $K(\Delta)$ must answer:

1. $N(\Delta_*) = 1 + 2a$, $\frac{dK}{d\Delta} = \frac{\Delta_*^{2a-1}}{4(1-a)}$ for $\Delta = \Delta_*$,
2. $\frac{dK}{d\Delta} = 0$ for $\Delta = 0$,
3. $\frac{dK}{d\Delta} \approx \frac{a}{\gamma^0 - 1} \cdot \Delta_*^{2a-1}$ for $\Delta \approx 0$

We shall claim at last that caloricity K is to satisfy the following conditions

$$K = K_K, \quad \frac{dK}{d\Delta} = 0 \quad \text{for } \Delta = 1$$

Let's define γ_0 . For this purpose we present P and E as the sum of heat and cold components

$$P = P_T(\rho, T) + P_X(\rho) \quad (14)$$

$$E = E_T(\rho, T) + E_X(\rho)$$

We consider P_T and E_T to have a simple form

$$P_T = (\gamma_0 - 1)\rho E_T \quad (15)$$

$$E_T = C_V T, \quad C_V = \text{const}$$

Using (15) we can write down (14) in a following way

$$PV = (\gamma_0 - 1)C_V T + f(V) \quad (16)$$

where

$$f(V) = V\varphi(V) + (\gamma_0 - 1)E_X(V), \quad V = 1/\rho$$

We shall take into account that DP is a mixture of gases under conditions of mechanical and thermal equilibrium, satisfying equation of state

$$PV_i = (\gamma_{oi} - 1)C_{vi}T + f_i(V_i) \quad (17)$$

Using additivity of V and E in the mixture

$$V = \sum_i \alpha_i V_i, \quad E = \sum_i \alpha_i E_i,$$

where α_i —mass concentration of i -th ingredient, we write down (14)–(17) as follows

$$\begin{cases} (\gamma_0 - 1)C_v T + f(V) = \\ \sum_i \alpha_i [(\gamma_{oi} - 1)C_{vi}T + f_i(V_i)] \\ C_v T + E_x(V) = \sum_i \alpha_i [C_{vi}T + E_{xi}(V_i)] \end{cases} \quad (18)$$

If $T=0$ then (18) involves connection between cold components of ingredients' equations of state and of mixture's one

$$f(V) = \sum_i \alpha_i f_i(V_i), \quad E_x(V) = \sum_i \alpha_i E_{xi}(V_i)$$

and so, for any $T \neq 0$ it must be

$$(\gamma_0 - 1)C_v = \sum_i (\gamma_{oi} - 1)\alpha_i C_{vi}, \quad C_v = \sum_i \alpha_i C_{vi}$$

Data on α_i , γ_i , C_{vi} for some explosives is given in refs. [1, 7, 8]. According to various theories, mass concentrations of DP ingredients differ. That's why we limited ourselves to consideration of averaged concentrations.

Within this approach the value of γ_0 , calculated as

$$\gamma_0 = \left[\sum_i (\gamma_{oi} - 1)\alpha_i C_{vi} \right] / \left[\sum_i \alpha_i C_{vi} \right]$$

for trotyl $\gamma_0 = 1.373$ is close to value $\gamma_0 = 1.378$ for hexogen. Since the difference is in the limits of averaging errors, so we take the common value $\gamma_0 = 1.374$ for trotyl and hexogen.

The procedures of numerical solving of Equation (11) couple with above-mentioned limitations on $K(\lambda)$ and $\gamma(\lambda)$ and simultaneous optimization of experimental data on $K(\lambda)$ and $K(\lambda)$ approximation are omitted because of their complexity. Obtained table dependencies are approximated with functions

$$\gamma = \begin{cases} \gamma_{\infty} & \text{for } x > 1 \\ \gamma_0 + (\gamma_{\infty} - \gamma_0) \cdot x(3 - 3x + x^2) & \text{for } x \leq 1 \end{cases}$$

$$\varphi = \begin{cases} \rho_{0i} D_i^2 \cdot A(x - 1)^m & \text{for } x > 1 \\ 0 & \text{for } x \leq 1 \end{cases}$$

where $x = \delta/\delta_0$, $A = 0.0139$, $\gamma_0 = 1.375$, $\gamma_{\infty} = 1.667$, $\delta_0 = 0.35$, $m = 2.284$.

To complete construction of DP equation of state for certain explosive it is necessary to determine only two parameters ρ_{0i} and D_i . Values of ρ_{0i} and D_i for compound explosives can be expressed through ρ_{0i} and D_i of ingredients in a simple way

$$\rho_{0i} = 1 / \left[\sum_i \alpha_i / \rho_{0i} \right], \quad D_i^2 = \sum_i \alpha_i D_{0i}^2,$$

where α_i —mass concentration of i -th ingredient. Values of ρ_{0i} and D_i for several explosives are listed in the table.

| Explosive | g/cm ³ | km/s |
|-----------|-------------------|------|
| Trotyl | 1.633 | 7.15 |
| Tetryl | 1.730 | 7.74 |
| PETN* | 1.770 | 8.35 |
| Hexogen | 1.820 | 8.80 |

* Pentaerythritol Tetranitrate

REFERENCES

1. F. A. Baum, K. P. Stanyukovich, B. I. Shekhter, Explosion Physics (Fizmatgiz, Moscow, 1959)
2. L. D. Landau, K. P. Stanyukovich, Dokl. AN USSR, 46 (1945) P. 399
3. K. Jukhanon, P. Person, Explosive Detonation (Mir, Moscow, 1973)
4. M. V. Zhernokletov, V. N. Zubarev, G. S. Telegin, Prikl. Mech. & Tekhn. Fiz., 4 (1969)
5. V. N. Zubarev, Prikl. Mech. & Tekhn. Fiz., 2 (1965)

6. N. M. Kuznetsov, K. K. Shvedov, *Fiz. Goren. & Vzryv.*, 4 (1965) p. 85
7. A. N. Dremin, S. D. Savrov, V. S. Trofimov, K. K. Shvedov, *Detonation Waves in Condensed Media* (Nauka, Moscow, 1970)
8. B. Ya. Svetlov, N. B. Yaremenko, *Theory and Properties of Industrial Explosives* (Nedra, Moscow, 1973)
9. R. Miller, in *Detonation And Two-Phase Flow*, Ed. S. S. Penner (Academic Press, New York, 1962)