PROPAGATION OF A STRONG DISCONTINUITY IN A BINARY MIXTURE OF GASES

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Strong discontinuities – shock-waves – arise in a continuous media under dynamic external loads. Simulation of their propagation in mixtures must take into account, for each mixture component, the mass, momentum, and energy conservation laws relating states of matter before and behind the shock front. This effort calculates the process of shock-wave propagation in a plane layer, i.e. a homogeneous mixture of two gases having different density. For this purpose, conservation laws, as well as quantities responsible for components interaction were numerically implemented. The Lagrangian stage of the Lagrangian – Eulerian calculations used the shock-wave computation method based on the solution of the system of nonlinear algebraic equations. Sensitivity of flow parameters to the cluster and pairwise interactions is investigated. Cluster interaction was shown to be the major contributor to velocities relaxation behind the strong discontinuity front. Profiles of thermodynamic quantities and mass velocities were obtained for each component.

Keywords: multicomponent media; conservation laws; shock wave; Lagrangian – Eulerian scheme; cluster interaction.

Introduction

In the one-dimensional case of a ideal continuous media consisting of N components, conservation laws of mass, momentum, and energy, as well as the equation of volume concentration of the i-th component in the Lagrangian coordinates for the plane symmetry in [1] are written as

$$\frac{d_i(\alpha_i \rho_i)}{dt} + (\alpha_i \rho_i)^2 \frac{\partial u_i}{\partial m_i} = 0, \tag{1}$$

$$\frac{d_i u_i}{dt} + \frac{\partial}{\partial m_i}\left(\alpha_i\left[P_i + F_i\right]\right) - \frac{R_i}{\rho_i} = 0, \tag{2}$$

$$\frac{d_i \varepsilon_i}{dt} + \frac{\partial}{\partial m_i}\left(\alpha_i\left[P_i u_i + F_i u_i + Q_i\right]\right) - \frac{\Phi_i + A_i}{\rho_i} = 0, \tag{3}$$

$$\frac{d_i \alpha_i}{dt} + \alpha_i \rho_i (U - u_i)\frac{\partial \alpha_i}{\partial m_i} + \Omega_i + B_i = 0, \tag{4}$$

where t is the time, x is the mass coordinate, α_i is the volume concentration of the i-th component, ρ_i is its density, u_i is its mass velocity, P_i is its pressure, $\varepsilon_i = E_i + 0.5u_i^2$ is the specific total energy, and E_i is the specific internal energy. In addition, the equations include terms that depend on other components, i.e. F_i is the force of cluster interaction between the i-th component and the mixture, R_i is the aggregate momentum of other components pairwise action upon the i-th component, Q_i is the flow energy directed to the

i-th component, Φ_i is energy exchange during pairwise interaction for the i-th component, A_i is the work of pairwise interaction for the i-th component, Ω_i is the function selected due to invariance of the equation of volume concentration to the Galilean transformation [2], B_i is the function that considers velocity exchange during pairwise interaction in the equation for volume concentration, U is the mass velocity of the mixture. $\frac{d_i}{dt} = \frac{\partial}{\partial t} + u_i \frac{\partial}{\partial x}$ the derivative along the trajectory of the i-th component (substantive).

The equation of state of the i-th component in the form

$$P_i = f_i(\rho_i, E_i)$$

is added to the system of equations (1)–(4), as well as the equation along the trajectory $\frac{d_i x}{dt} = u_i$.

The state of matter in the i-th component before and behind the shock front is described by the following system of nonlinear algebraic equations

$$\alpha_{i+} = \alpha_{i-}, \tag{5}$$

$$\frac{1}{\rho_{i+}} W_i + u_{i+} = \frac{1}{\rho_{i-}} W_i + u_{i-}, \tag{6}$$

$$u_{i+} W_i - P_{i+} - F_{i+} = u_{i-} W_i - P_{i-} - F_{i-}, \tag{7}$$

$$\varepsilon_{i+} W_i - u_{i+}(P_{i+} + F_{i+}) - Q_{i+} = \varepsilon_{i-} W_i - u_{i-}(P_{i-} + F_{i-}) - Q_{i-}, \tag{8}$$

where W_i is the velocity of shock-wave propagation in the i-th component, index '$-$' is the state of matter before the shock front and index '$+$' – behind the shock front. Quantities before the discontinuity are assumed to be known. Jump in entropy across the shock-wave is associated with the energy dissipation mechanism. There are 4 different energy dissipation mechanisms [3]. We will use the method given in [4] and apply it to the multicomponent media. A sequence of equations is added in order to close the system of equations (5)–(8). One equation among them is the equation of state [5]:

$$P_{i+} = f_i(\rho_{i+}, E_{i+}). \tag{9}$$

In this equation, the specific internal energy E_{i+} is expressed in terms of ε_{i+} and u_{i+} as

$$E_{i+} = \varepsilon_{i+} - 0.5 u_{i+}^2. \tag{10}$$

Another two equations are expressions for functions of cluster interaction, F_{i+} and Q_{i+}:

$$F_{i+} = -0.5 \rho_{i+}(u_{i+} - U_+)^2, \tag{11}$$

$$Q_{i+} = 0.5 \rho_{i+}(U_+ - u_{i+})(P_{i+} + \rho_{i+} E_{i+}). \tag{12}$$

The mass velocity of the mixture, U_+ is expressed in terms of components velocities:

$$U_+ = \sum_{i=1}^{N} \eta_i u_{i+}, \tag{13}$$

where N is the number of components. Expressions for the mass concentration of i-th component and density of the mixture are defined as follows

$$\eta_{i+} = \alpha_{i+} \rho_{i+} / \rho_+, \tag{14}$$

$$\rho_+ = \sum_{i=1}^{N} \alpha_i \rho_{i+}. \tag{15}$$

The relation between the velocity of shock-wave propagation in the mixture (W) and the appropriate velocity in the i-th component closes the system of equations:

$$W_i = W\left(\frac{\rho_{i-}C_{i-}}{\rho_- C_-}\right). \tag{16}$$

1. Description of the Difference Scheme

Practical experience of solving problems with multicomponent media [3] demonstrates that two-stage methods, wherein the first stage uses the difference scheme of computation in the Lagrangian coordinates, erase a number of difficulties and simplify the computational procedure. Consider the structure of any one of two-stage methods. At the first stage, the grid is interpreted to be the Lagrangian one. Any known method of computation in the Lagrangian coordinates is used to determine auxiliary quantities, u_i^*, P_i^*, α_i^*, as well as main quantities at the time t^{n+1}. Here, the Eulerian coordinates of cell boundaries x_n are varying and take on values x_j^{n+1}. At the second stage, a new grid is generated and the grid x_j^{n+1} is re-interpolated onto this new grid. Re-interpolation is performed in accordance with the instantaneous $(t = const)$ laws of mass, momentum, and energy conservation. The new grid can coincide with but also can differ from the initial Eulerian grid x_j^n (moving Eulerian grid).

The divergence difference scheme developed by V. F. Kuropatenko [3] is used as the two-stage method. The mesh pattern is given in Fig. 1. All grid quantities are determined in the center of grid cells on the Lagrangian coordinate m. Boundary between cells at $m = m_i$ has the Eulerian coordinate x_i^n. At the time $t^{n+1} = t^n + \tau$ the Lagrangian coordinate m_i remains the same but the Eulerian coordinate of the point is changing and takes on the value x_i^{n+1}.

1.1. The Lagrangian Stage

Evaluation of auxiliary quantities According to [4], it is necessary to find the auxiliary quantities P_i^*, α_i^*, u_i^*. All cells are grouped into two types, i.e. cells wherein matter of i-th component is compressed $((u_i)_{j+1}^n - (u_i)_j^n < 0)$ and cells wherein grid rarefaction wave is present $((u_i)_{j+1}^n - (u_i)_j^n \geq 0)$. For the first-type cells, the system of equations (5)–(16) is resolved and such quantities of this system as the volume concentration, pressure, and velocity behind the shock front are taken as auxiliary quantities. For the second-type cells, equations (1)–(3) are approximated by difference equations.

Let us write out difference equations in order to find velocity, pressure, and volume concentration of the i-th component in the cell rarefaction wave

$$(u_i)_k^* = (u_i)_k^n - \frac{\tau}{2}\left[\frac{(\alpha_i)_{k+0.5}^n\left((P_i)_{k+0.5}^n + (F_i)_{k+0.5}^n\right)}{(h_i)_k^n} - \right.$$
$$\left. - \frac{(\alpha_i)_{k-0.5}^n\left((P_i)_{k-0.5}^n + (F_i)_{k-0.5}^n\right)}{(h_i)_k^n} + \frac{(R_i)_k^n}{(\rho_i)_k^n}\right],$$

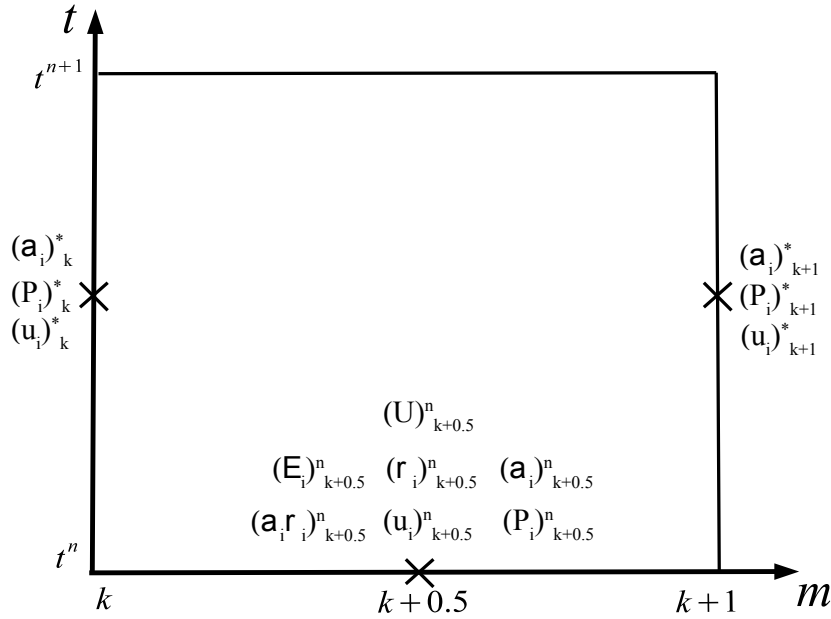**Fig. 1**. Schematic distribution of quantities in a cell

$$(P_i)_k^* = (P_i)_k^n - \frac{\tau}{2}\left[\left(\frac{\partial P_i}{\partial \rho_i}\right)_{E_i}\right]_k^n \left((\rho_i^2)_k^n \frac{(\alpha_i)_k^n\left((u_i)_{k+0.5}^n - (u_i)_{k-0.5}^n\right)}{(h_i)_k^n} - (\rho_i)_k^n \frac{(\Omega_i)_k^n + (B_i)_k^n}{(\alpha_i)_k^n} + \right.$$

$$\left. + \frac{(U_k^n - (u_i)_k^n)\left((\alpha_i)_{k+0.5}^n - (\alpha_i)_{k-0.5}^n\right)}{(h_i)_k^n}\right) - \frac{\tau}{2}\left[\left(\frac{\partial P_i}{\partial E_i}\right)_{\rho_i}\right]_k^n \left(\frac{(R_i)_k^n(u_i)_k^n - (\Phi_i)_k^n - (A_i)_k^n}{(\rho_i)_k^n} + \right.$$

$$\left. + \frac{(\alpha_i)_k^n\left[(P_i)_k^n + (F_i)_k^n\right]\left((u_i)_{k+0.5}^n - (u_i)_{k-0.5}^n\right)}{(h_i)_k^n} + \frac{(\alpha_i)_{k+0.5}^n(Q_i)_{k+0.5}^n - (\alpha_i)_{k-0.5}^n(Q_i)_{k-0.5}^n}{(h_i)_k^n}\right),$$

where τ is the iteration step in time, h_i is the iteration step in space and $(\rho_i^2)_k^n = [(\rho_i)_k^n]^2$. The equation for the volume concentration is written out as

$$(\alpha_i)_k^* = (\alpha_i)_k^n - \frac{\tau}{2}\left[\frac{(\alpha_i)_k^n(\rho_i)_k^n\left(U_k^n - (u_i)_k^n\right)\left((\alpha_i)_{k+0.5}^n - (\alpha_i)_{k-0.5}^n\right)}{(h_i)_k^n} + (\Omega_i)_k^n + (B_i)_k^n\right].$$

Here, the following notation is introduced

$$\left[\left(\frac{\partial P_i}{\partial \rho_i}\right)_{E_i}\right]_k^n = \frac{1}{2}\left(\left[\left(\frac{\partial P_i}{\partial \rho_i}\right)_{E_i}\right]_{k-0.5}^n + \left[\left(\frac{\partial P_i}{\partial \rho_i}\right)_{E_i}\right]_{k+0.5}^n\right) =$$

$$= \frac{P_i\left((\rho_i)_{k-0.5}^n + \delta, (E_i)_{k-0.5}^n\right) - P_i\left((\rho_i)_{k-0.5}^n, (E_i)_{k-0.5}^n\right)}{2\delta} +$$

$$+ \frac{P_i\left((\rho_i)_{k+0.5}^n + \delta, (E_i)_{k+0.5}^n\right) - P_i\left((\rho_i)_{k+0.5}^n, (E_i)_{k+0.5}^n\right)}{2\delta},$$

$$\left[\left(\frac{\partial P_i}{\partial E_i}\right)_{\rho_i}\right]^n_k = \frac{1}{2}\left(\left[\left(\frac{\partial P_i}{\partial E_i}\right)_{\rho_i}\right]^n_{k-0.5} + \left[\left(\frac{\partial P_i}{\partial E_i}\right)_{\rho_i}\right]^n_{k+0.5}\right) =$$

$$= \frac{P_i\left((\rho_i)^n_{k-0.5}, (E_i)^n_{k-0.5} + \delta\right) - P_i\left((\rho_i)^n_{k-0.5}, (E_i)^n_{k-0.5}\right)}{2\delta} +$$

$$+ \frac{P_i\left((\rho_i)^n_{k+0.5}, (E_i)^n_{k+0.5} + \delta\right) - P_i\left((\rho_i)^n_{k+0.5}, (E_i)^n_{k+0.5}\right)}{2\delta},$$

where $\delta << 1$.

Evaluation of main quantities. At the second stage, we find main quantities determined in middles of grid cells. As this takes place, cell boundaries change as

$$x^{n+1}_k = x^n_k + \tau(u_i)^*_k,$$

and masses and partial densities in cells depend on the problem symmetry. Therefore, in the plane case $(h_i)^n_{k+0.5} = (\alpha_i\rho_i)^n_{k+0.5}(x^n_{k+1} - x^n_k)$,

$$(\alpha_i\rho_i)^{n+1}_{k+0.5} = \frac{(h_i)^n_{k+0.5}}{x^{n+1}_{k+1} - x^{n+1}_k}.$$

The velocity at the next time step is

$$(u_i)^{n+1}_{k+0.5} = (u_i)^n_{k+0.5} - \tau\left(\frac{(\alpha_i)^*_{k+1}\left[(P_i)^*_{k+1} + (F_i)^n_{k+1}\right] - (\alpha_i)^*_k\left[(P_i)^*_k + (F_i)^n_k\right]}{(h_i)^n_{k+0.5}} - \frac{(R_i)^n_{k+0.5}}{(\rho_i)^n_{k+0.5}}\right).$$

The specific internal energy is found from the formula

$$(E_i)^{n+1}_{k+0.5} = (E_i)^n_{k+0.5} - \tau\left[\frac{\left((\alpha_i)^*_{k+1}\left[(u_i)^*_{k+1}\left((P_i)^*_{k+1} + (F_i)^n_{k+1}\right) + (Q_i)^n_{k+1}\right]\right.}{(h_i)^n_{k+0.5}} - \right.$$

$$\left. - \frac{(\alpha_i)^*_k\left[(u_i)^*_k\left((P_i)^*_k + (F_i)^n_k\right) + (Q_i)^n_k\right]}{(h_i)^n_{k+0.5}} - \frac{(A_i)^n_{k+0.5} + (\Phi_i)^n_{k+0.5}}{(\rho_i)^n_{k+0.5}}\right] -$$

$$- 0.5\left((u^2_i)^{n+1}_{k+0.5} - (u^2_i)^n_{k+0.5}\right).$$

The volume concentration is found from the formula

$$(\alpha_i)^{n+1}_{k+0.5} = (\alpha_i)^n_{k+0.5} - \tau\left[(\alpha_i)^n_{k+0.5}(\rho_i)^n_{k+0.5}\left(U^n_{k+0.5} - (u_i)^n_{k+0.5}\right)\frac{(\alpha_i)^*_{k+1} - (\alpha_i)^*_k}{(h_i)^n_{k+0.5}} + \right.$$

$$\left. + (\Omega_i)^n_{k+0.5} + (B_i)^n_{k+0.5}\right].$$

The rest quantities are independent of the problem symmetry. Here, the following notation is also introduced:

$$(\rho_i)^{n+1}_{k+0.5} = \frac{(\alpha_i\rho_i)^{n+1}_{k+0.5}}{(\alpha_i)^{n+1}_{k+0.5}},$$

$$(P_i)^{n+1}_{k+0.5} = f_i((\rho_i)^{n+1}_{k+0.5}, (E_i)^{n+1}_{k+0.5}),$$

$$(F_i)_{k+0.5}^{n+1} = -0.5(\rho_i)_{k+0.5}^{n+1} \left(U_{k+0.5}^{n+1} - (u_i)_{k+0.5}^{n+1}\right)^2,$$

$$(Q_i)_{k+0.5}^{n+1} = \frac{1}{2}(U_{k+0.5}^{n+1} - (u_i)_{k+0.5}^{n+1})((P_i)_{k+0.5}^{n+1} + (\rho_i)_{k+0.5}^{n+1}(E_i)_{k+0.5}^{n+1}),$$

$$(R_i)_{k+0.5}^{n+1} = (\alpha_i)_{k+0.5}^{n+1} \sum_{j=1}^{N}(\alpha_j)_{k+0.5}^{n+1}(R_{ij})_{k+0.5}^{n+1} = (\alpha_i)_{k+0.5}^{n+1} \sum_{j=1}^{N}(\alpha_j)_{k+0.5}^{n+1}a_{ij}((u_j)_{k+0.5}^{n+1} - (u_i)_{k+0.5}^{n+1}),$$

$$(\Phi_i)_{k+0.5}^{n+1} = (\alpha_i)_{k+0.5}^{n+1} \sum_{j=1}^{N}(\alpha_j)_{k+0.5}^{n+1}(\Phi_{ij})_{k+0.5}^{n+1} = (\alpha_i)_{k+0.5}^{n+1} \sum_{j=1}^{N}(\alpha_j)_{k+0.5}^{n+1}b_{ij}((P_j)_{k+0.5}^{n+1} - (P_i)_{k+0.5}^{n+1}),$$

$$(\Omega_i)_{k+0.5}^{n+1} = (\alpha_i)_{k+0.5}^{n+1} \sum_{j=1}^{N}(\alpha_j)_{k+0.5}^{n+1} \frac{b_{ij}((P_j)_{k+0.5}^{n+1} - (P_i)_{k+0.5}^{n+1})}{((c_i)_{k+0.5}^{n+1} + (c_j)_{k+0.5}^{n+1})((\rho_i)_{k+0.5}^{n+1}(c_i)_{k+0.5}^{n+1} + (\rho_j)_{k+0.5}^{n+1}(c_j)_{k+0.5}^{n+1})},$$

$$(A_i)_{k+0.5}^{n+1} = (\alpha_i)_{k+0.5}^{n+1} \sum_{j=1}^{N}(\alpha_j)_{k+0.5}^{n+1}a_{ij} \frac{((u_j)_{k+0.5}^{n+1})^2 - ((u_i)_{k+0.5}^{n+1})^2}{2},$$

$$(B_i)_{k+0.5}^{n+1} = (\alpha_i)_{k+0.5}^{n+1} \sum_{j=1}^{N}(\alpha_j)_{k+0.5}^{n+1}d_{ij}a_{ij} \frac{((u_j)_{k+0.5}^{n+1})^2 - ((u_i)_{k+0.5}^{n+1})^2}{2},$$

where $c_i = \sqrt{\frac{P_i}{\rho_i^2}\left(\frac{\partial P_i}{\partial E_i}\right)_{\rho_i} + \left(\frac{\partial P_i}{\partial \rho_i}\right)_{E_i}}$, or in the difference form

$$(c_i)_{k-0.5}^{n} = \sqrt{\frac{(P_i)_{k-0.5}^{n}}{(\rho_i^2)_{k-0.5}^{n}}\left[\left(\frac{\partial P_i}{\partial E_i}\right)_{\rho_i}\right]_{k-0.5}^{n} + \left[\left(\frac{\partial P_i}{\partial \rho_i}\right)_{E_i}\right]_{k-0.5}^{n}}.$$

1.2. The Eulerian Stage

After the auxiliary and main quantities for each component are determined on the Lagrangian grid, it is necessary to convert quantities from the "old" grid (obtained after the Lagrangian stage) to the "new" grid (unified three-dimensional grid).

Let a cell in the new grid of size Δx has L old grid cells wherein the solution is already found, i.e. quantities Δx_k, $(\rho_i)_k$, $(u_i)_k$, $(P_i)_k$ are known. Note that one part of the old grid cells wholly fall within the cell and the other part – only partially. It is important that values Δx_k satisfy the condition:

$$\Delta x = \sum_{k=1}^{L} \Delta x_k.$$

Masses of old grid cells and also the mass of the Eulerian cells are determined from the following equations

$$(\Delta M_i)_k = (\rho_i)_k \Delta x_k,$$

$$\Delta M = \sum_{k=1}^{L} \Delta M_k.$$

Let us introduce the volumetric and mass concentrations

$$\alpha_k = \frac{\Delta x_k}{\Delta x}, \qquad \xi_k = \frac{\Delta M_k}{\Delta M}.$$

It follows that $(\alpha_i)_k$, $(\rho_i)_k$ satisfy the conditions

$$\sum_{k=1}^{L} \alpha_k = 1, \qquad \sum_{k=1}^{L} \xi_k = 1.$$

Obeying the mass, momentum, and energy conservation laws, we derive equations to determine quantities of the averages in the new cell

$$\rho_i = \sum_{k=1}^{L} \alpha_k (\rho_i)_k, \qquad u_i = \sum_{k=1}^{L} \xi_k (u_i)_k, \qquad E_i = \sum_{k=1}^{L} \xi_k \left(E_k + 0.5((u_i)_k - U)^2 \right).$$

2. Statement of the Problem

Let us consider the test problem on the steady shock-wave propagation in a homogeneous mixture of two polyatomic ideal gases (with the equation of state $P = (\gamma - 1)\rho E$). Suppose this mixture is in the enclosed volume and each gas is in the state of rest. Pressure of each component is assumed to be equal to zero, volume concentrations of gases are equal in value, $\gamma_1 = \gamma_2 = \frac{5}{3}$. Density of a light gas equals 1 and density of a heavy gas equals 2. A boundary condition is established, i.e. a rigid wall moving at rate 1 (see Fig. 2).

$$U_{\mathsf{LB}} = 1 \qquad U_{\mathsf{L}} = U_{\mathsf{H}} = 0, \qquad P_{\mathsf{L}} = P_{\mathsf{H}} = 0, \qquad U_{\mathsf{RB}} = 0$$
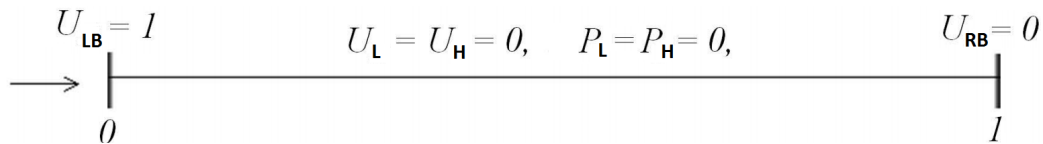
Fig. 2. Statement of the problem. Parameters of the light gas are denoted by letter L and parameters of the heavy gas – by letter H.

3. Results of Numerical Simulation

The method presented in Section 1 was used to perform computations in the above problem. Our calculations used different values of parameters a_{12} and b_{12}, responsible for pairwise interaction. Consider different situations in calculations with cluster interaction that is universal and independent of the components' state.

3.1. Only Cluster Interaction $(a_{12} = 0,\ b_{12} = 0)$

Figures 3–5 show profiles of density, pressure, and velocity at the time $t = 0.5$.

These results obviously demonstrate that cluster interaction equalizes mass velocities of components. Thus, it makes no sense to increase velocity relaxation behind the shock front through increment of the parameter a_{12} in the case of pairwise interaction against cluster interaction. Hence, we must increment the parameter b_{12} in order to estimate contribution of pairwise interaction against cluster interaction.
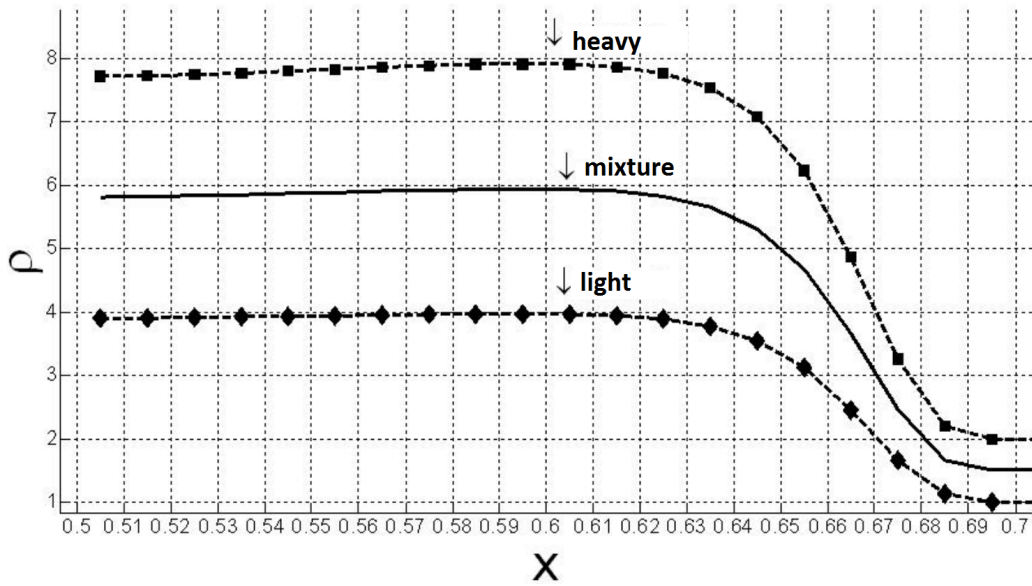
Fig. 3. Density profiles for the light gas, the heavy gas, and their mixture
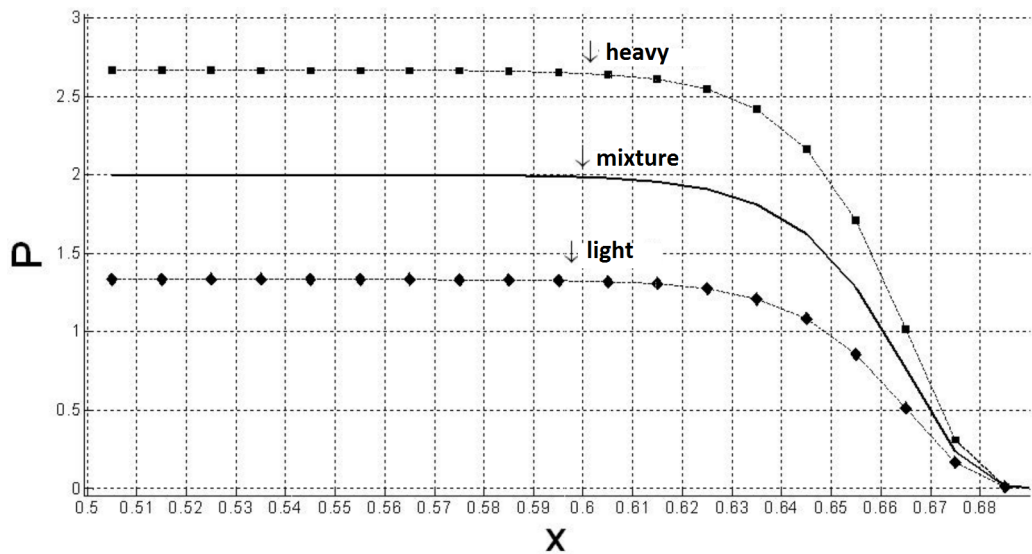

Fig. 4. Pressure profiles for the light gas, the heavy gas, and their mixture

3.2. Pairwise Interaction in the Presence of Cluster Interaction, Pressure Exchange ($a_{12} = 0$, $b_{12} = 10$)

Figures 6–8 show profiles of density, pressure, and velocity at the time $t = 0.5$.

It is easily seen that notable increment of the parameter b_{12} does not result in significant pressure relaxation behind the shock front. Also, mass velocities are noted to differ in the vicinity of the front, i.e. the light component is accelerating first and then it urges the heavy component to increase its velocity.
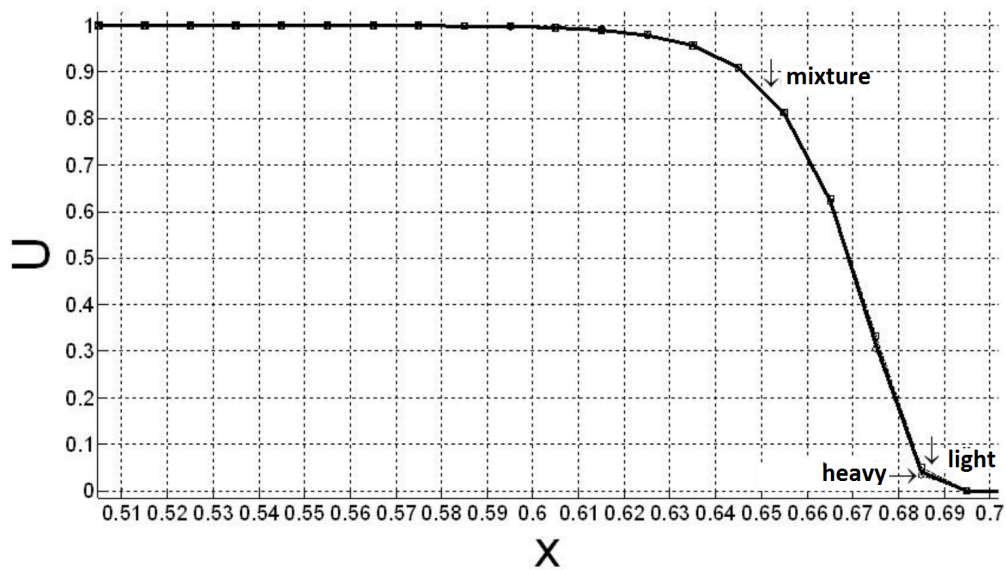
Fig. 5. Mass velocity profiles for the light gas, the heavy gas, and their mixture
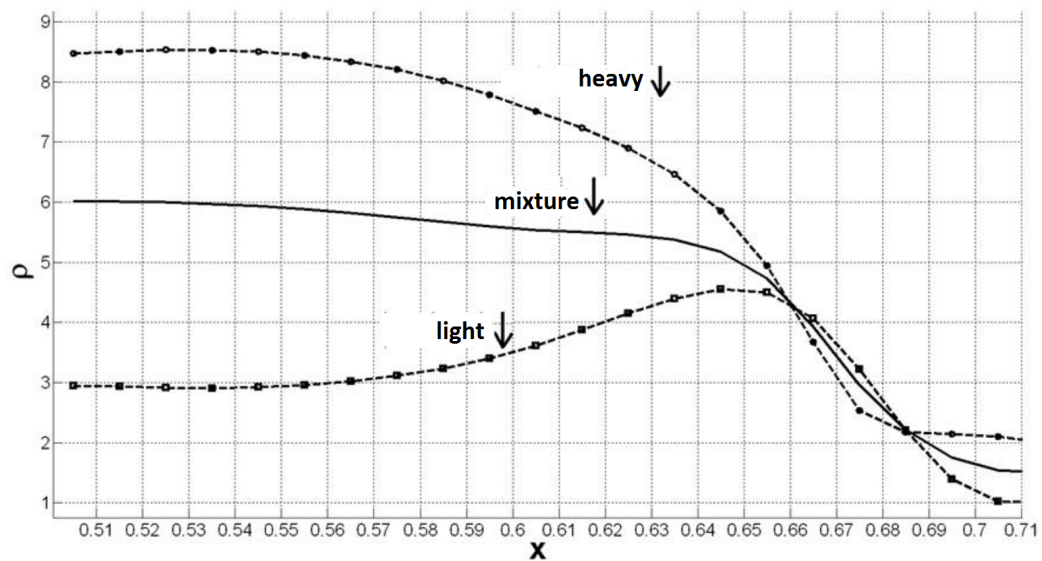

Fig. 6. Density profiles for the light gas, the heavy gas, and their mixture

Conclusion

The method for determining auxiliary quantities, which is based on the conservations laws on the strong discontinuity in mixtures, is developed. One-dimensional calculations of the shock-wave propagation in the plane layer of two gases are performed. The numerical solution proves to be adequate as for variation of physical quantities. Different values of component interaction parameters are considered. Flow parameters are shown to depend on cluster interaction.
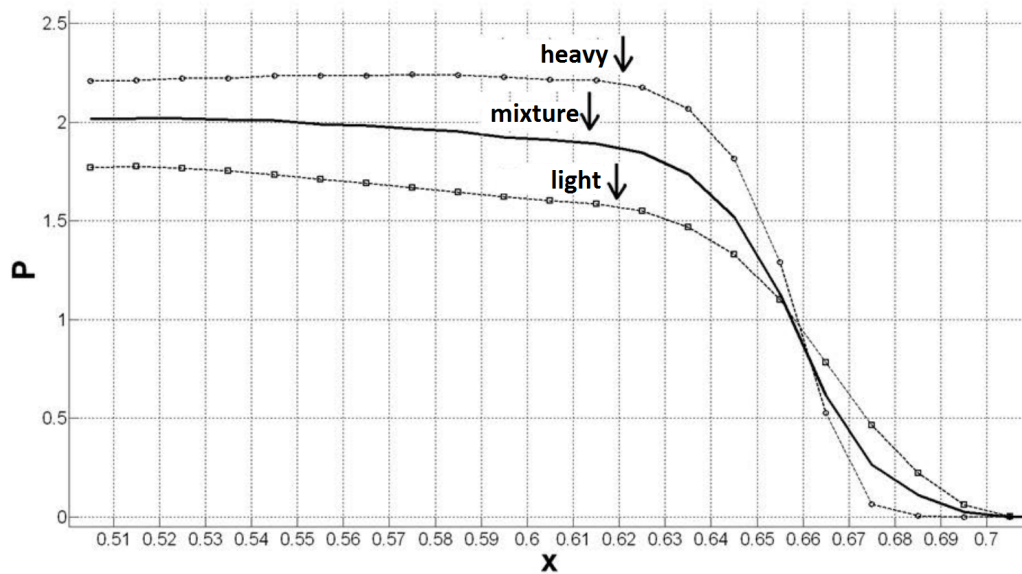
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Fig. 7. Pressure profiles for the light gas, the heavy gas, and their mixture
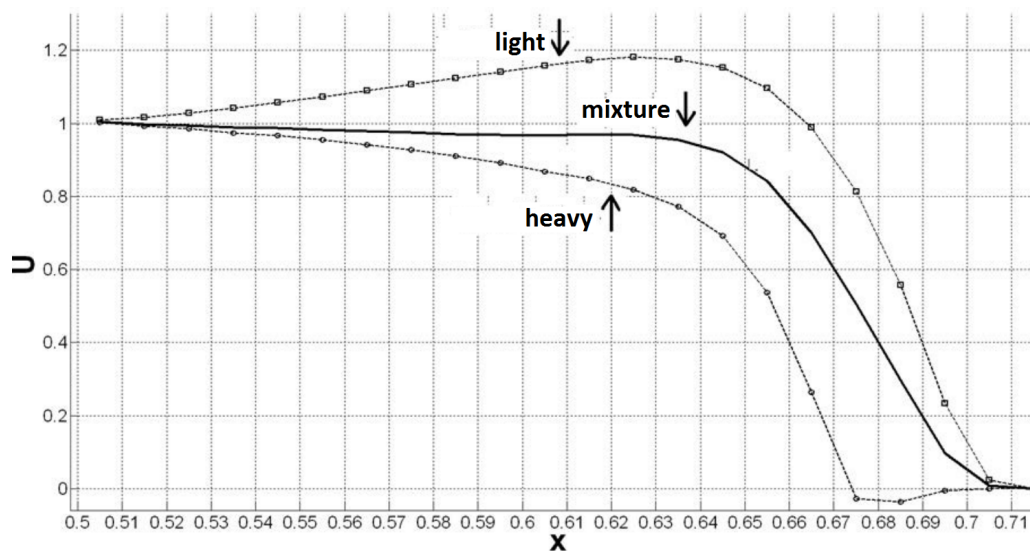

Fig. 8. Mass velocity profiles for the light gas, the heavy gas, and their mixture

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О РАСПРОСТРАНЕНИИ СИЛЬНОГО РАЗРЫВА В ДВУХКОМПОНЕНТНОЙ СМЕСИ ГАЗОВ

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Сильные разрывы – ударные волны возникают в сплошной среде при динамических внешних воздействиях. При моделировании их распространения в смесях веществ для каждого из компонентов необходимо учитывать законы сохранения массы, импульса и энергии, связывающие состояния вещества перед поверхностью разрыва и за ней. В данной работе рассчитывается процесс распространения ударной волны в плоском слое – гомогенной смеси двух газов различной плотности. Для этого численно реализовывались законы сохранения и величины, отвечающие за взаимодействие компонентов. Расчеты выполнялись лагранжево-эйлеровым методом. На лагранжевом этапе был использован метод расчета ударных волн, основанный на решении системы нелинейных алгебраических уравнений. Исследовано влияние кластерного и парного взаимодействий на параметры течения. Показано, что наибольший вклад в релаксацию скоростей за фронтом сильного разрыва дает именно кластерное взаимодействие. При различных параметрах взаимодействия получены профили термодинамических величин и массовых скоростей для каждого из компонентов.

Ключевые слова: многокомпонентная среда; законы сохранения; ударная волна; лагранжево-эйлерова схема; кластерное взаимодействие.

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