Journal of Applied Mechanics and Technical Physics, Vol. 46, No. 1, pp. 1–8, 2005

MOMENTUM AND ENERGY EXCHANGE IN NONEQUILIBRIUM MULTICOMPONENT MEDIA

V. F. Kuropatenko

UDC 532.529

In multicomponent media, the equilibrium states are defined by thermodynamic equilibrium conditions in the form of the equalities between the pressures and temperatures of the components, the maximum entropy principle (or the free-energy minimum) for the mixture, and the equality of the velocities of the components. The conservation laws for the components allow for their interaction with each other in the form of forces and energy fluxes containing the differences of the velocities, pressures, and temperatures of the components. The form of momentum and energy exchange between each component and the continuum expressing the collective properties of the ensemble of components, is also considered. It is shown that these momentum and energy fluxes are different from zero only for the states of multicomponent media with velocity nonequilibrium.

Key words: *equilibrium, multiphase multicomponent medium, interaction of components, closure of the system of equations.*

Introduction. The great diversity of multicomponent media of natural and man-made origins makes their investigation an extremely difficult problem. Dynamic processes in multicomponent media, accompanied by phase transitions of separate components, are especially complex. The components move at different velocities, which leads to variations in their concentrations in the four-dimensional space x_1, x_2, x_3, t. Because of the interaction of the components, a nonequilibrium multicomponent medium reaches an equilibrium state after a certain relaxation time. The relaxation processes of pressures, temperatures, and velocities in multicomponent media have been studied both for particular mixtures and in general formulations [1–7]. In these studies, the sum of the functions defining the momentum and energy exchange between all components was usually set equal to zero. This created difficulties in determining the increase in the entropy of multicomponent media during relaxation. Accounting for nonequilibrium kinetic energy partly eliminates these difficulties.

Below, we consider problems that arise in the development of models for continuum based on the hypothesis of interacting continua [1]. In these models, the components are structural elements of multicomponent media, which are present simultaneously at each point of the volume. Averaging operations using their characteristics make it possible to pass from the characteristics of the components to the characteristics of a certain continuum, which, like the characteristics of the components, are continuous in the four-dimensional space x_1, x_2, x_3, t. Therefore, they can be described by conservation laws. A continuum whose characteristics are obtained by averaging the corresponding characteristics of the mixture components will be referred to as a virtual continuum. Obviously, one of the conditions of equivalence between multicomponent media and corresponding virtual continua is a macrolevel manifestation of the interaction of components in the multicomponent medium. In other words, the conservation laws of a virtual continuum necessarily depend on the conservation laws of the components.

Zababakhin Institute of Technical Physics, Snezhinsk 456770; v.f.kuropatenko@vniitf.ru. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 46, No. 1, pp. 7–15, January–February, 2005. Original article submitted March 2, 2004.

The main requirements to which models of multicomponent media should satisfy are as follows:

1. Each component with number i ($i = 1, 2, \dots, N$) is characterized by the volumetric and mass concentrations (α_i and η_i, respectively), which satisfy the conditions

$$\sum_{i=1}^{N} \alpha_i = 1, \qquad \sum_{i=1}^{N} \eta_i = 1. \tag{1}$$

2. For each ith component, there is a complete set of characteristics — the density ρ_i, pressure P_i, temperature T_i, internal energy E_i, entropy S_i, velocity $\boldsymbol{U}_i$, kinetic energy $K_i = 0.5|\boldsymbol{U}_i|^2$, total energy $\varepsilon_i = E_i + K_i$, etc.

3. For each component, the stress tensor is split into a spherical part and a deviator.

4. The thermodynamic quantities (P_i, ρ_i, E_i, T_i, S_i, etc.) are related by equations of state. The modern equations of state [8] describe polymorphic phase transitions, melting, vaporization, and ionization, considerably extending the range of applicability of models of multicomponent media.

5. If the values of P, T and $\boldsymbol{U}$ for components with numbers i and j differ, then momentum and energy exchange occurs between the components.

Continuum of a Component. We shall follow the well-studied approach [2–7] to describing the laws of conservation of components. It is assumed that when multiplied by the volumetric concentration, the specific physical quantities (in unit volume of the ith component) become continuous in the volume occupied by the mixture. They include both the main quantities ($\alpha_i\rho_i$, $\alpha_i P_i$, $\alpha_i\rho_i\boldsymbol{U}_i$, $\alpha_i\rho_i E_i$, $\alpha_i\rho_i K_i$, $\alpha_i\rho_i S_i$, and $\alpha_i\rho_i T_i$) and a number of other combinations of specific (in unit volume) parameters.

We next consider a mixture of media ignoring turbulence, heat conduction, the effect of fields, and chemical reactions. These assumptions for ideal media lead to the simplest laws of conservation of mass, momentum, and energy for the ith component:

$$\frac{\partial}{\partial t}(\alpha_i\rho_i) + \nabla\alpha_i\rho_i\boldsymbol{U}_i = 0; \tag{2}$$

$$\frac{\partial}{\partial t}(\alpha_i\rho_i\boldsymbol{U}_i) + \frac{\partial}{\partial x_k}(\alpha_i\rho_i U_{ik}\boldsymbol{U}_i) + \nabla\alpha_i P_i = \alpha_i\boldsymbol{R}_i; \tag{3}$$

$$\frac{\partial}{\partial t}(\alpha_i\rho_i\varepsilon_i) + \nabla(\alpha_i\boldsymbol{U}_i(P_i + \rho_i\varepsilon_i)) = \alpha_i\Phi_i. \tag{4}$$

System (2)–(4) is supplemented by the equation of state for the ith component

$$P_i = P_i(\rho_i, E_i) \tag{5}$$

and the equations for the parameters $\boldsymbol{R}_i$ and Φ_i that describe the rate of momentum and energy exchange between the ith component and the remaining components.

We shall consider multicomponent media with possible nonequilibrium with respect to the parameters P, T and $\boldsymbol{U}$. This implies that there are three functions τ_P, τ_T, and τ_U that describe the relaxation times of pressure, temperature, and velocity and depend on the parameters of the interacting components. In real physical processes, the values of τ_P, τ_T, and τ_U are finite. There are, however, a great number of papers devoted to the so-called asymptotic models of multicomponent media, in which all or some of the relaxation times are set equal to zero or infinity. In the present paper, such models are not considered.

Since a multicomponent media is in nonequilibrium, the equations of motion and energy should contain the forces and fluxes generated by particular kinds of nonequilibria, according to the theory of nonequilibrium processes [9]. From this point of view, Eqs. (3) and (4) need to be refined. They include the single force P_i — the spherical part of the stress tensor. One kind of interaction between the components is friction. Therefore, the force F_i exerted on the ith component by the multicomponent medium is taken in the most general form and is considered a tensor. Equation (3) contains the vector $\boldsymbol{R}_i$, which defines the momentum exchange between the ith component and the remaining components of the multicomponent medium. The form of the vector $\boldsymbol{R}_i$ is well justified, but the addition of the tensor force F_i extends the capabilities of the model.

Equation (4) contains the function Φ_i, which defines the energy exchange between the ith component and the remaining components of the multicomponent medium due to nonequilibrium in pressure and temperatures.